



Flächenschwerpunkt zusammengesetzter Körper

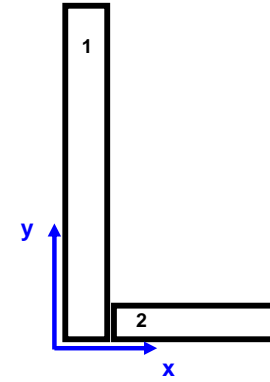
gelb: Eingabefelder
 hellgelb: es sind die Formelwerte für Rechteckquerschnitte hinterlegt, ggf. ändern
 Ausschnitte: als negative Flächen eingeben

Beispiel:

Teil Nr	Breite	Höhe	Schwerpunkt in x / y		A _i	x _{si} * A _i	y _{si} * A _i
	b _i	h _i	x _{si}	y _{si}			
1	8,0	100,0	4,0	50,0	800,0	3.200,0	40.000,0
2	42,0	8,0	29,0	4,0	336,0	9.744,0	1.344,0
Summen					1.136,0	12.944,0	41.344,0

x_s = 11,4

y_s = 36,4



Berechnung der Flächenschwerpunktskoordinaten

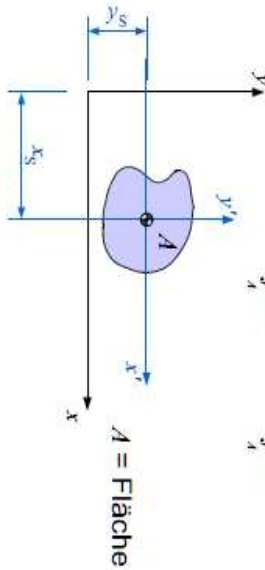
$$x_s = \frac{\int_A x \, dA}{\int dA} \Leftrightarrow x_s = \frac{\sum_{i=1}^n x_i \cdot A_i}{\sum_{i=1}^n A_i}$$

$$y_s = \frac{\int_A y \, dA}{\int dA} \Leftrightarrow y_s = \frac{\sum_{i=1}^n y_i \cdot A_i}{\sum_{i=1}^n A_i}$$

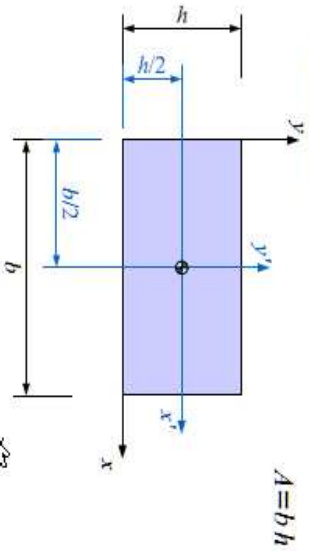
Schwerpunktlage von Flächen (Winkel im Bogenmaß)

Definition

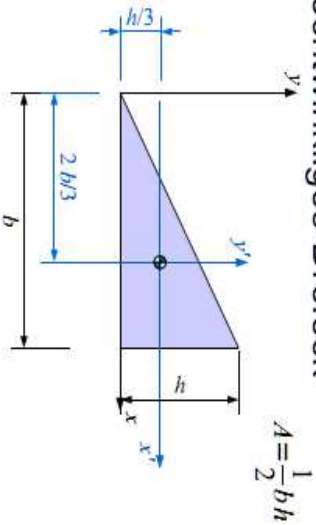
$$x_S = \frac{\int x \, dA}{\int dA} \quad y_S = \frac{\int y \, dA}{\int dA}$$



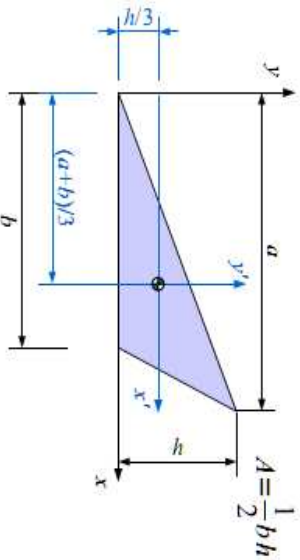
Rechteck



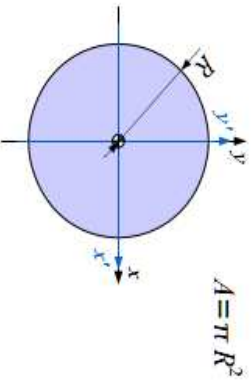
rechtwinkliges Dreieck



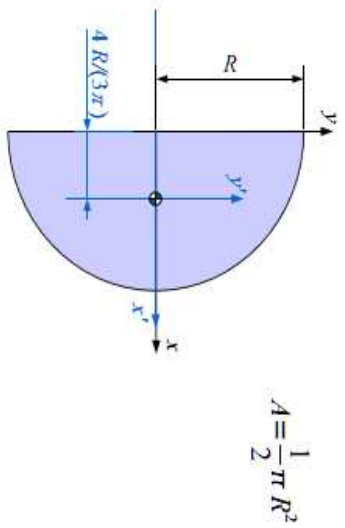
Dreieck



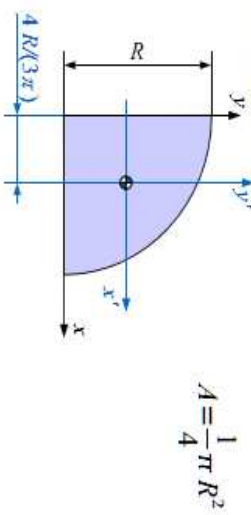
Kreis



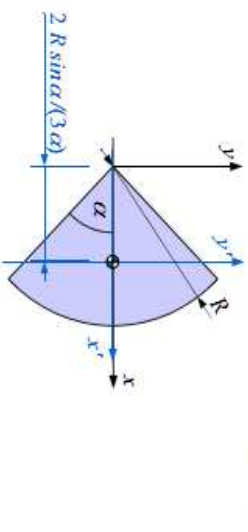
Halbkreis



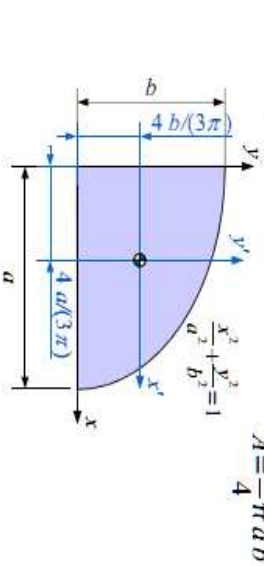
Viertelkreis



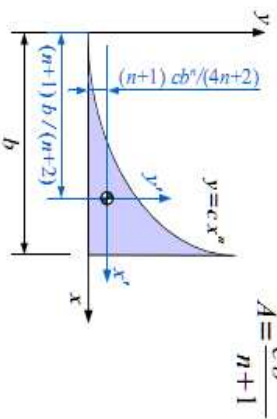
Kreissector

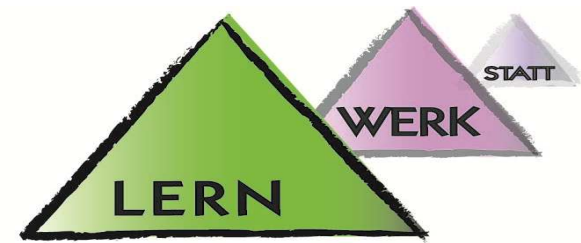


Viertelellipse



Potenzfunktion





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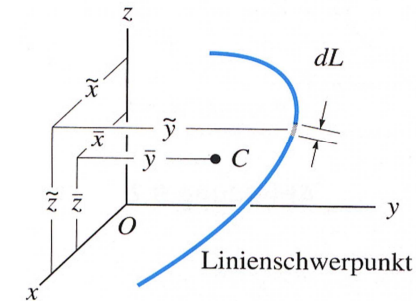
Linien­schwer­punkt zusam­men­ge­setz­ter Linien

gelb: Eingabefelder
 Fehlende Linie: als negative Länge eingeben

Beispiel:

Teil Nr	Schwerpunkt in x / y / z				$x_{si} \cdot L_i$	$y_{si} \cdot L_i$	$z_{si} \cdot L_i$
	x_{si}	y_{si}	z_{si}	L_i			
1	4,0	50,0	10,0	100,0	400,0	5.000,0	1.000,0
2	29,0	4,0	20,0	50,0	1.450,0	200,0	1000
Summen				150,0	1.850,0	5.200,0	2.000,0

$x_s = 12,3$
 $y_s = 34,7$
 $z_s = 0,0$



Berechnung der Linien­schwer­punkt­skoordinaten

$$x_s = \frac{\int_L x \, dL}{\int dL} \Leftrightarrow x_s = \frac{\sum_{i=1}^n x_i \cdot L_i}{\sum_{i=1}^n L_i}$$

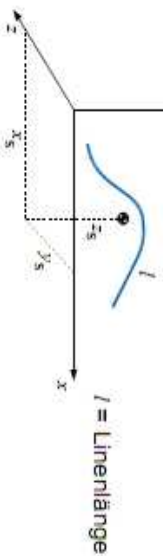
$$y_s = \frac{\int_L y \, dL}{\int dL} \Leftrightarrow y_s = \frac{\sum_{i=1}^n y_i \cdot L_i}{\sum_{i=1}^n L_i}$$

$$z_s = \frac{\int_L z \, dL}{\int dL} \Leftrightarrow z_s = \frac{\sum_{i=1}^n z_i \cdot L_i}{\sum_{i=1}^n L_i}$$

Schwerpunktlage von Linien (Winkel im Bogenmaß)

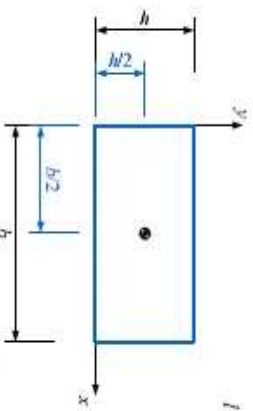
Definition

$$x_s = \frac{\int x \, dl}{\int dl} \quad y_s = \frac{\int y \, dl}{\int dl} \quad z_s = \frac{\int z \, dl}{\int dl}$$



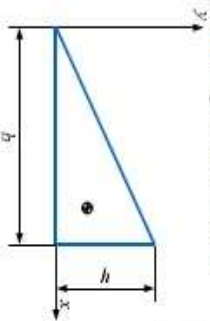
Rechteck

$$l = 2(b + h)$$



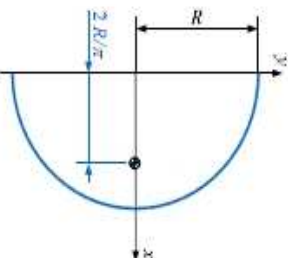
rechtwinkliges Dreieck

$$l = b + h + \sqrt{b^2 + h^2}$$



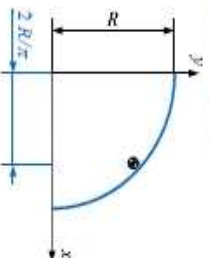
Halbkreis

$$l = \pi R$$



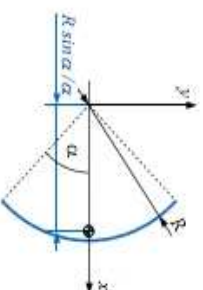
Viertelkreis

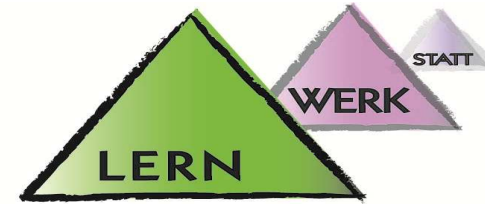
$$l = \frac{1}{2} \pi R$$



Kreissector

$$l = 2\alpha R$$





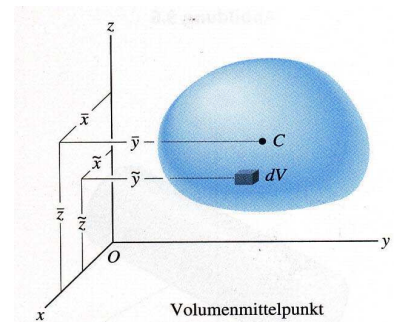
Volumenschwerpunkt zusammengesetzter Körper

gelb: Eingabefelder
 Fehlendes Volumen (Loch): als negatives Volumen eingeben

Beispiel:

Teil Nr	Mittelpunkt in x / y / z				$x_{si} \cdot V_i$	$y_{si} \cdot V_i$	$z_{si} \cdot V_i$
	x_{si}	y_{si}	z_{si}	V_i			
1	4,0	50,0	20,0	100,0	400,0	5.000,0	2.000,0
2	29,0	4,0	23,0	50,0	1.450,0	200,0	1.150,0
Summen				150,0	1.850,0	5.200,0	3.150,0

$x_s = 12,3$
 $y_s = 34,7$
 $z_s = 21,0$



Berechnung der Volumenschwerpunktskoordinaten

$$x_s = \frac{\int_V x \, dV}{\int_V dV} \Leftrightarrow x_s = \frac{\sum_{i=1}^n x_i \cdot V_i}{\sum_{i=1}^n V_i}$$

$$y_s = \frac{\int_V y \, dV}{\int_V dV} \Leftrightarrow y_s = \frac{\sum_{i=1}^n y_i \cdot V_i}{\sum_{i=1}^n V_i}$$

$$z_s = \frac{\int_V z \, dV}{\int_V dV} \Leftrightarrow z_s = \frac{\sum_{i=1}^n z_i \cdot V_i}{\sum_{i=1}^n V_i}$$



Masseschwerpunkt zusammengesetzter Körper

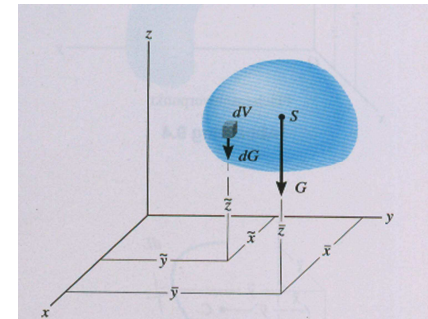
gelb: Eingabefelder

Fehlendes Volumen (Loch): als negatives Volumen eingeben

Beispiel:

Teil Nr	Schwerpunkt in x / y / z					m _i	ρ _i ·x _{si} ·V _i	ρ _i ·y _{si} ·V _i	ρ _i ·z _{si} ·V _i
	x _{si}	y _{si}	z _{si}	ρ _i	V _i				
1	4,0	50,0	20,0	1,0	100,0	100,0	400,0	5.000,0	2.000,0
2	29,0	4,0	23,0	2,0	50,0	100,0	2.900,0	400,0	2.300,0
Summen						200,0	3.300,0	5.400,0	4.300,0

x_s= 16,5
 y_s= 27,0
 z_s= 21,5



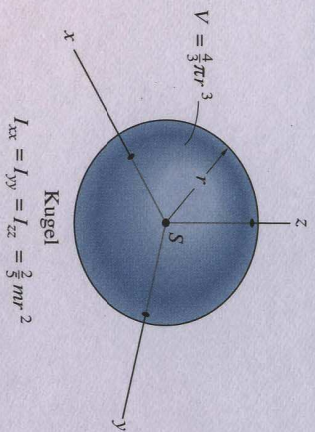
Berechnung der Massenschwerpunktskoordinaten

$$x_s = \frac{\int_V x \cdot \rho \, dV}{\int \rho \, dV} \Leftrightarrow x_s = \frac{\sum_{i=1}^n x_i \cdot \rho_i \cdot V_i}{\sum_{i=1}^n \rho_i \cdot V_i}$$

$$y_s = \frac{\int_V y \cdot \rho \, dV}{\int \rho \, dV} \Leftrightarrow y_s = \frac{\sum_{i=1}^n y_i \cdot \rho_i \cdot V_i}{\sum_{i=1}^n \rho_i \cdot V_i}$$

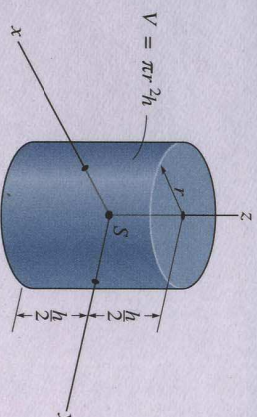
$$z_s = \frac{\int_V z \cdot \rho \, dV}{\int \rho \, dV} \Leftrightarrow z_s = \frac{\sum_{i=1}^n z_i \cdot \rho_i \cdot V_i}{\sum_{i=1}^n \rho_i \cdot V_i}$$

Schwerpunkt und Massenträgheitsmoment homogener Körper



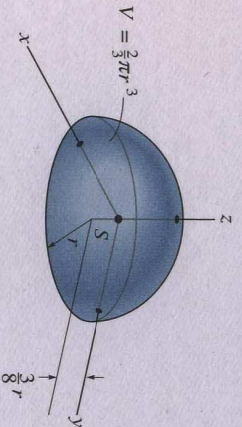
$$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} m r^2$$

Kugel



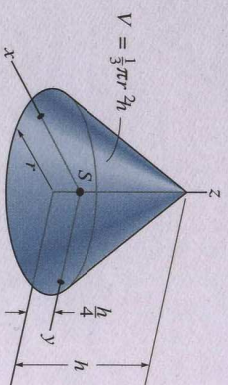
$$I_{xx} = I_{yy} = \frac{1}{12} m (3r^2 + h^2) \quad I_{zz} = \frac{1}{2} m r^2$$

Zylinder



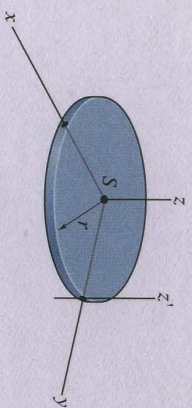
$$I_{xx} = I_{yy} = 0,259 m r^2 \quad I_{zz} = \frac{2}{3} m r^2$$

Halbkugel



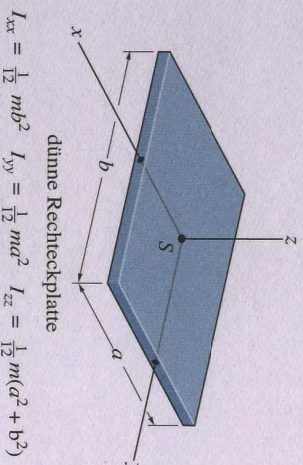
$$I_{xx} = I_{yy} = \frac{3}{80} m (4r^2 + h^2) \quad I_{zz} = \frac{3}{10} m r^2$$

Kegel



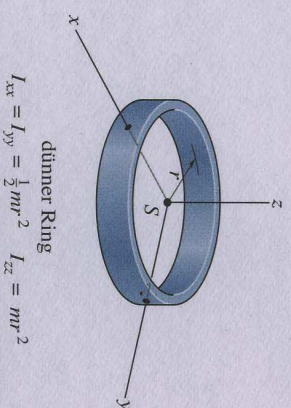
$$I_{xx} = I_{yy} = \frac{1}{4} m r^2 \quad I_{zz} = \frac{1}{2} m r^2 \quad I_{z'z'} = \frac{3}{2} m r^2$$

dünne Kreisscheibe



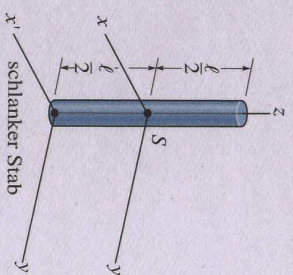
$$I_{xx} = \frac{1}{12} m b^2 \quad I_{yy} = \frac{1}{12} m a^2 \quad I_{zz} = \frac{1}{12} m (a^2 + b^2)$$

dünne Rechteckplatte



$$I_{xx} = I_{yy} = \frac{1}{2} m r^2 \quad I_{zz} = m r^2$$

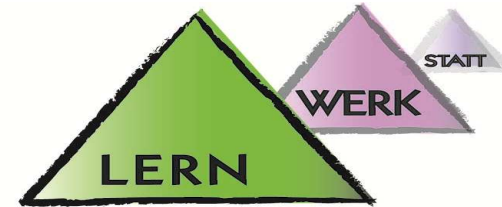
dünner Ring



$$I_{xx} = I_{yy} = \frac{1}{12} m l^2 \quad I_{x'x'} = I_{y'y'} = \frac{1}{3} m l^2 \quad I_{z'z'} = 0$$

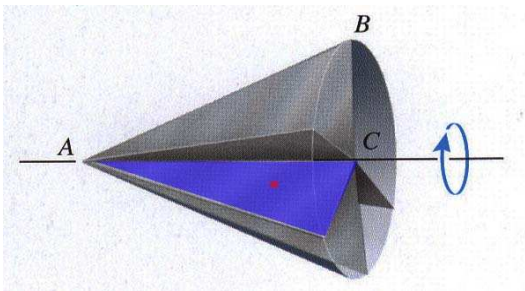
schlanker Stab

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Volumen und Oberfläche von Rotationskörpern

Volumen= erzeugende Fläche * Bogenlänge des Schwerpunkts um Drehachse



Oberfläche= erzeugende Linie * Bogenlänge des Schwerpunkts um Drehachse

